

FREQUENCY DESIGN OF BUS TRANSIT IN MIXED URBAN TRANSPORT NETWORK

(都市の混合交通ネットワークにおけるバス交通の頻度に関する分析)

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1. Introduction

Recently, many researchers have made supreme effort in designing an efficient surface public transport networks as mass transit is currently losing its market share to the private modes despite its important role in people's daily commute. Existing studies have been concentrated on improving the configuration of transit line and improving frequency. Frequency design which is the focus of this study, has gained a lot of interest from researchers; however, only a very few studies considered the traveler's response in a satisfactory manner (1, 2, 3). Nevertheless, these studies are restricted to the analysis of the case where the transit demand is fixed and the in-vehicle travel time is constant.

It should be noted that while several types of public transport share the same road space with general traffic, which naturally consists of various kinds of vehicles (e.g. large and small, fast and slow), the road congestion should be taken into account in the transit frequency design. Moreover, travelers could have a variety of travel modes when conducting a trip. For this reason, these existing studies might provide a misleading implication in real world application, especially when the networks exhibit many modes having strongly different performance. To my knowledge, LeBlanc⁽⁴⁾ is the only literature that consider the elastic demand of bus transit while the road congestion is assumed to have smaller impact on bus user's travel time than on that of private mode user. It could be observed especially when mixed traffic is congested, bus cannot move faster than other modes. Instead, it is the smaller-size mode (e.g. car, motorbike) that can move faster thanks to their flexibility

and ability to maneuver in narrow space. Therefore, the previous study by LeBlanc might not be able to represent the case of mixed traffic network appropriately.

In this study, bus transit frequency design with variable demand, under fixed total demand, is proposed by explicitly take into account the interaction between different vehicles in mixed traffic of the urban road network, targeting bus, car, and motorbike.

2. Notations and Assumptions

For simplicity, the notations that will be used throughout this paper are presented as follow: $G = (N, A)$ is a network with node set N and link set A . Let \tilde{m}, \bar{m}, m refers to private and para-transit mode, public transit mode, and all mode respectively. \bar{A} corresponds to the set of link that belong to feasible transit path from origin node i to destination node j . A_i^+ is set of link a starting from origin node i outwards. q_{ij}, q_{ij}^m are total trip demand of an OD pair ij and that for each mode m respectively. $f_{ij,r}^m, h_{ij,r}^m$ refer to the user path flow and vehicle path flow of mode m on path r from i to j respectively. $V_{a,\bar{m}}$ is the link flow of passenger of transit mode \bar{m} , and $v_{a,\tilde{m}}$ is the link flow of private and para-transit vehicle mode \tilde{m} ($v_{a,\tilde{m}} = V_{a,\bar{m}} / \lambda_{\tilde{m}}$). λ_m, γ_m denote the occupancy rate and passenger car equivalent factor of mode m respectively. τ, φ^m are parameter of logit model and mode attractiveness. $c_{ij,r}^m, \mu_{ij}^m$ refer to the generalized travel cost of mode m on path r and the minimum travel cost of mode m to travel from i to j respectively. $t_{a,\tilde{m}}, t_{a,\bar{m}}$ are in-vehicle travel time on link a of private mode and public transit, while $t_{a,\tilde{m}}^o, t_{a,\bar{m}}^o, C_{a,\tilde{m}}, C_{a,\bar{m}}$ denote their corresponding free flow

travel time and link capacity. $\alpha, \beta, \eta, \phi$ are non-negative parameters of the link performance function. $\bar{F}_{a,\bar{m}}, K_{a,\bar{m}}$ are the fixed public transit flow and public transit link capacity (boarding capacity) on link a . f_l, f_a are the frequency of public transit line l and the total link frequency of link a . $T_{ij,r}^m, FARE_{ij,r}^m, W_{ij,r}^m$ are in-vehicle travel time, fare, and waiting time of mode m on path r traveling between i - j .

Before proceeding, the following fundamental assumptions are adopted: 1). The total O-D demand is known and constant 2). All travelers have access to all travel modes available in the multi-modal network (e.g. car, motorbike, para-transit, bus) 3). Bus routes are defined a priori (they are not subject to change) 4). Bus frequency is regarded as a continuous variable 5). Flat fare system is adopted (same fare for one trip) 6). Walking to bus stop and transfer time is not considered 7). Inter-arrival of bus at a stop is constant and the arrival rate of passenger at bus stop is considered to be uniform. 8). Bus passenger waiting at the bus stop (node i) will board the first arriving bus that will go along feasible paths associated with O-D pair i - j . 9). All intermediate nodes are duplicated to represent bus stops.

3. Model Formulation

In order to design bus line frequency, transit demand is computed by a combined mode split assignment model in which modal split and mixed traffic's link flow are computed with respect to the change of bus frequency.

3.1. Combined modal split assignment model

A) Disutility function:

The disutility of using mode m to travel from origin i to destination j is characterized by the sum of generalized cost and the specific constant related to each modes, and it can be written as: $U_{ij}^m = -\tau(c_{ij,r}^m - \varphi^m)$.

The term generalized cost can be computed from: $c_{ij,r}^m = T_{ij,r}^m + FARE_{ij,r}^m + W_{ij,r}^m$. As flat fare for one trip is

adopted, the indices ij and r will be dropped, and $FARE^m$ will be used throughout this paper without loss of generality.

It is also worth to note that fare and waiting time for private mode users are assumed to be zero. For para-transit users, it is assumed that waiting time is zero. So only the generalized cost of traveling by bus takes the three components into account of its generalized travel cost. Under the assumption 7 and 8, the index r will be drop from the average waiting time for bus, and it can be computed as:

$$W_{ij}^m = \frac{\omega}{\sum_{a \in \bar{A}_i^+} f_a}, \forall a \in \bar{A}_i^+ \quad \omega = 0.5 \quad [1]$$

B) Formulation of link travel time:

Under mixed traffic, the actual travel time for any private mode and non-schedule para-transit user on link a is a function of the summation of all flow on that link including the fixed bus flow: $t_{a,\bar{m}} = t_{a,\bar{m}}(\sum_{\bar{m}} v_{a,\bar{m}}, \bar{F}_{a,\bar{m}})$. Since we adopt the link performance function developed by Bureau of Public Roads (BPR), the in-vehicle travel time of a private mode and para-transit is:

$$t_{a,\bar{m}} = t_{a,\bar{m}}^o \left[1 + \alpha \left(\frac{\sum_{\bar{m}} v_{a,\bar{m}} \cdot \gamma_{\bar{m}} + \bar{F}_{a,\bar{m}} \cdot \gamma_{\bar{m}}}{C_{a,\bar{m}}} \right)^\beta \right] \quad [2]$$

Since bus vehicle shares the same road space with other modes, the travel time of traveling on link a depends on the vehicle flow of the road network as well as passenger flow on the transit service such that: $t_{a,\bar{m}} = t_{a,\bar{m}}(\sum_{\bar{m}} v_{a,\bar{m}}, \bar{F}_{a,\bar{m}}, V_{a,\bar{m}})$. It is fully expressed as:

$$t_{a,\bar{m}} = t_{a,\bar{m}}^o \left[1 + \alpha \left(\frac{\sum_{\bar{m}} v_{a,\bar{m}} \cdot \gamma_{\bar{m}} + \bar{F}_{a,\bar{m}} \cdot \gamma_{\bar{m}}}{C_{a,\bar{m}}} \right)^\beta \right] + \eta \left(\frac{V_{a,\bar{m}}}{K_{a,\bar{m}}} \right)^\phi \quad [3]$$

C) Equilibrium solution for mixed traffic

Assume that user route choice is governed by Wardrop's principle; therefore, equilibrium path flow

should satisfy the following conditions:

$$\begin{cases} (c_{ij,r}^m - \mu_{ij}^m)h_{ij,r}^m = 0 \\ (c_{ij,r}^m - \mu_{ij}^m) \geq 0 \end{cases}, \forall m, r, ij \quad [4]$$

Moreover, Mode split function is characterized by a multinomial logit function such that:

$$q_{ij}^m = q_{ij} \frac{e^{-\tau(\mu_{ij}^m - \varphi^m)}}{\sum_n e^{-\tau(\mu_{ij}^n - \varphi^n)}}, \forall m, ij \quad [5]$$

It should be noted that the unequal impact on congestion produced by different vehicle in mixed traffic results in asymmetric cost function, so there is no equivalent mathematical program whose solution represents the equilibrium flow. We adopt “diagonalization algorithm”, which is based on solving iteratively a series of standard UE programs, to find the equilibrium flow of the mixed traffic network and the bus network. Assume that at n^{th} iteration, link flows of all modes are known. Then the following mathematical problem can be formulated:

$$\begin{aligned} \min z^{(n)} = & \sum_{\bar{m}} \sum_a \int_0^{V_{a,\bar{m}}} t_{a,\bar{m}}(\omega, \bar{F}_{a,\bar{m}}) d\omega \\ & + \sum_{\bar{m}} \sum_a \int_0^{V_{a,\bar{m}}} t_{a,\bar{m}}(v_{a,\bar{m}}, \bar{F}_{a,\bar{m}}, \omega) d\omega \\ & + \sum_m \sum_{ij} W_{ij}^m q_{ij}^m + \sum_m \sum_{ij} FARE^m q_{ij}^m \\ & + \sum_m \left[\frac{1}{\tau} \sum_{ij} q_{ij}^m (\ln q_{ij}^m - 1) - \sum_{ij} \varphi^m q_{ij}^m \right] \end{aligned} \quad [6]$$

s.t.

$$\sum_r f_{ij,r}^m = q_{ij}^m, \quad \forall m, ij \quad [7]$$

$$\sum_m q_{ij}^m = q_{ij}, \quad \forall ij \quad [8]$$

$$f_{ij,r}^m \geq 0, \quad \forall m, r, ij \quad [9]$$

3.2. Frequency design model

The objective of this study is to minimize the total travel cost of multi-modal network users as well as the operating cost converted from frequency. Hence, the frequency design model is expressed implicitly as follow:

$$\begin{aligned} \min Z(\mathbf{f}) = & \sum_a \sum_m t_{a,m}(V_{a,m}(\mathbf{f}), \mathbf{f}) \cdot \mathbf{V}(\mathbf{f}) + \sum_{ij} \sum_m q_{ij}^m W_{ij}^m \\ & + \sum_{ij} \sum_m q_{ij}^m (FARE^m / VOT) + \theta \sum_l f_l \cdot F_l \end{aligned} \quad [10]$$

$$s.t. \quad f_l \geq 1 \quad [11]$$

The first term, second term, and third term of Eq.[10] represents the users' total travel time, waiting time, and fare of all modes at a given frequency respectively; while the last term refers to the operating cost caused by frequency setting. The constraint $f_l \geq 1$ ensures that at least there is one bus departs per period of study.

4. Algorithm for Solving the Problem

In this model, the most demanding part is at finding equilibrium for mixed traffic network. The complete simulation process can be briefly summarized below:

Step 1: initialize a feasible line frequency vector \mathbf{f}_l that satisfies the constraint in the frequency design model, and initialize feasible link flow and O-D trip demand

Step 2: given \mathbf{f}_l , the modal-split assignment model solves for the equilibrium flow of all modes on the mixed traffic and produces equilibrium link flow \mathbf{V}^* .

Step 3: the equilibrium link flow \mathbf{V}^* and the feasible frequency \mathbf{f}_l are inserted into objective function. The objective function is then evaluated.

Step 4: Hooke and Jeeves algorithm⁽⁵⁾ is used to find a new frequency of each bus line, and then return to step 2.

This procedure repeats until a better frequency that reduces the cost of the system with comparing to the previous iteration is not found.

5. Numerical Example

In order to check the operation of the model and the algorithm proposed in this paper, a numerical example is conducted based on sensitivity analysis using the simple network.

5.1. Strategy of the sensitivity analysis

The strategy of the sensitivity analysis used in this subsection is described as follow:

- Convergence check of the objective function with different initial frequencies.
- Effect of OD demand changes on optimal frequencies
- Effect of changes in operating cost per unit of frequency on the optimal frequency
- Effect of changes of bus capacity on the optimal frequency as well as OD travel time.
- Effect of changes of car occupancy rate on the optimal bus frequency as well as OD travel time.

5.2. Test network

It is assumed that four traffic modes are operated on this network: car (mode1), motorbike (mode2), Motodop (mode3), and bus (mode4). Let bus and Motodop adopt the system of flat fare, $FARE = 50$ monetary unit per trip, and assume that there are three bus lines operating on this network. The capacity of each bus vehicle is set to 20 passengers. The cost per frequency setting is set to $F_l = 10^4$ and factor converting cost caused by frequency setting to time $\theta = 1$.

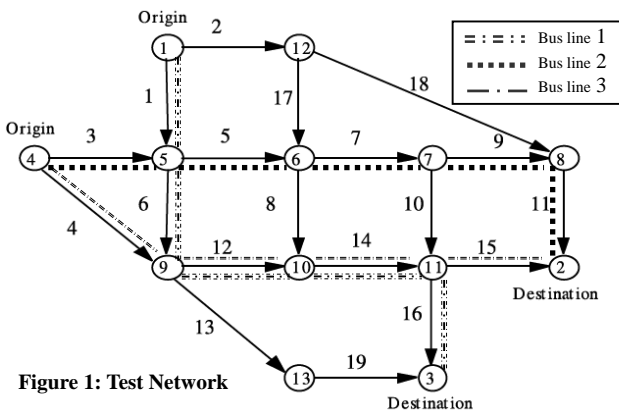


Figure 1: Test Network

Table 1: O-D trip table

From/To	2	3
1	1000	800
4	1000	800

Table 2: Parameters used in simulation

Parameter	Modes			
	1	2	3	4
Parameter of logit model τ	0.1	0.1	0.1	0.1
Mode attractiveness φ^m	0.0	-25.0	-25.0	50.0
PCE factor γ_m	1.0	0.5	0.5	3.0
Occupancy rate λ_m	1.0	1.0	1.0	--

Table 3: Link performance characteristics:

Link No.	$t_{a,m}^o$ (min)			$C_{a,m}$ (veh/hour)		
	Mode 1	Mode 2, 3	Mode 4	Mode 1	Mode 2, 3	Mode 4
1	5	8	5	300	350	290
2	5	8	5	300	350	290
3	5	8	5	300	350	290
4	9	10	9	300	350	290
5	5	8	5	300	350	290
6	5	8	5	300	350	290
7	5	8	5	300	350	290
8	5	8	5	300	350	290
9	5	8	5	300	350	290
10	5	8	5	300	350	290
11	5	8	5	300	350	290
12	5	8	5	300	350	290
13	7	8	7	300	350	290
14	5	8	5	300	350	290
15	5	8	5	300	350	290
16	5	8	5	300	350	290
17	5	8	5	300	350	290
18	12	20	12	300	350	290
19	5	8	5	300	350	290

5.3. Result and discussion

The computational result of the optimal frequencies was determined using various values of initial frequencies randomly selected for each line of bus transit. Table 4 shows the, different initial frequencies converge similar value of the optimal frequencies. Therefore, it could be believed that the solution may be close to the global solution of the problem. Figure 2 illustrates the convergence process of the of the optimization problem with the initial value $(f_1, f_2, f_3) = (1, 1, 1)$. The result of the optimal frequencies is, therefore, found at $(f_1^*, f_2^*, f_3^*) = (4.66, 3.00, 3.00)$ with the objective function $C^*(\mathbf{f}) = 373009$.

Table 4: Optimal frequency result from different initial frequencies

Initial Frequency	f_1	f_2	f_3
(1, 1, 1)	4.66	3.00	3.00
(1, 2, 3)	4.66	3.00	3.00
(2, 4, 3)	4.66	3.00	3.00
(5, 4, 2)	4.66	3.00	3.00
(1, 6, 2)	4.61	3.12	3.12
(2, 1, 4)	4.61	3.12	3.12

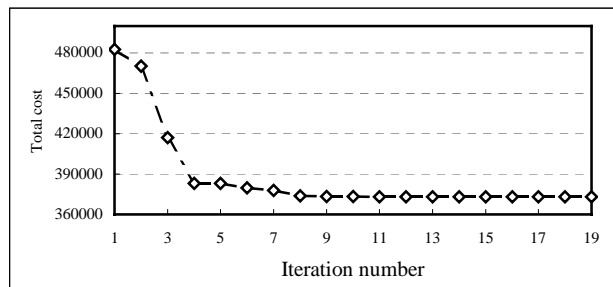


Figure 2: Convergence process of the optimization problem

In the analyses below, the effect of various parameters on the transit optimal frequencies is investigated. It is found that the optimal frequencies of bus lines considerably increase as the OD demands become higher. Figure 3 indicates that an increase from 0% to 60% of the demand of OD (1-2) and (1-3) has resulted in the rise of frequencies from 0% to 66.9% on line 1, from 0% to 30.9% on line 2, and from 0% to 34.9% on line 3 respectively. This result highlights the importance of the periodical modification of transit frequency in order to cope with the increase of travel demand resulting from urban population growth. The effect of variation of operation cost on the optimal frequency is also observed enabling us to identify the necessity for decreasing the frequency when the operation cost rises in order to keep the system performance optimum. The result in figure 4 shows that the optimal frequency of bus line 2 and line 3 remain stable up to 40% increase of cost per unit frequency, while the frequency of line 1 shows significant decrease within this period. However, further increase of operation cost slightly affects the optimal frequency of line 1 and line 3, while that of line 2 critically declines.

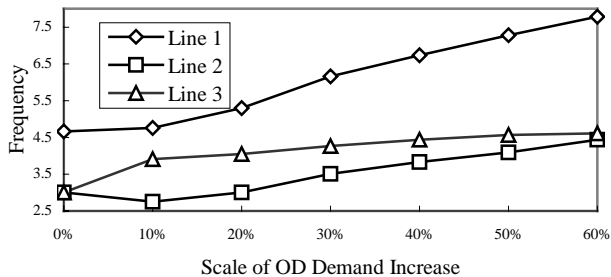


Figure 3: Optimal frequencies vs. OD demand fluctuation

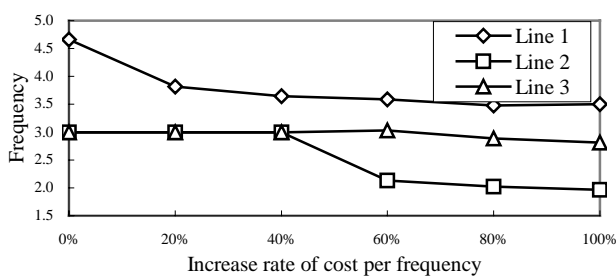


Figure 4: Optimal frequencies vs. increase of cost per frequency

Regarding the capacity of bus vehicle, the result in figure 5-(a), 5-(b), and 5-(c) show that using buses able to carry more passengers will reduce the line frequencies, improve the OD travel time, and somehow increase (or at least maintain) the number of bus users respectively. This finding underscores the importance of bus capacity selection appropriate for servicing in the mixed traffic network.

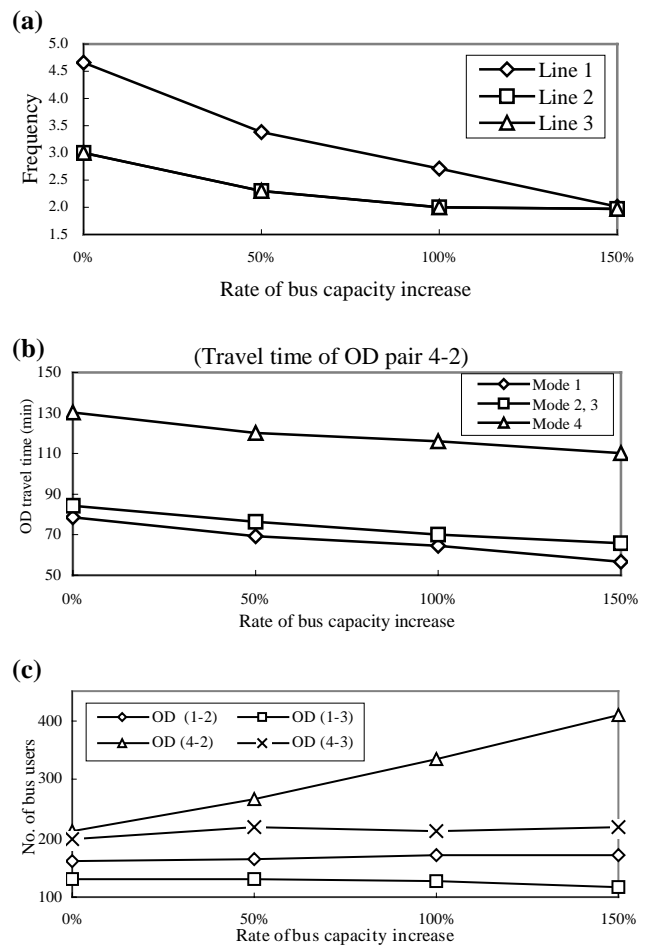


Figure 5: (a). Optimal frequency vs. bus capacity
(b). Travel time OD (4-2) vs. bus capacity
(c). Number of bus users vs. bus capacity

It is also discovered that when car occupancy rate increases, the bus line frequencies decrease, and the number of bus passengers significantly decreases as clearly shown in figure 6-(a), 6-(b), and 6-(c). From the practical viewpoint, this result gives an important implication on how the increase of vehicle occupancy can positively contribute to reducing the travel time for the

whole network, which might lead to the idea of promoting carpooling in the urban congested network. However, this policy tends to have adverse effect on the transit market as it reduces the number of bus users.

travel time. The result indicates that the optimal frequencies might be close to the global optimum as different initial frequencies converge the same value. The sensitivity analyses also suggest the necessity of periodical modification of the transit line frequency to cope with the increase in travel demand as well as the variation of the operation cost. Moreover, the analysis was also extended to give an overview of two options that could be the effective measures for improving the travel time, i.e. bus capacity selection and carpooling.

However, the results above depend strongly on the parameters setting in the test network. Therefore, further extension to the application in the real network should be addressed in future study.

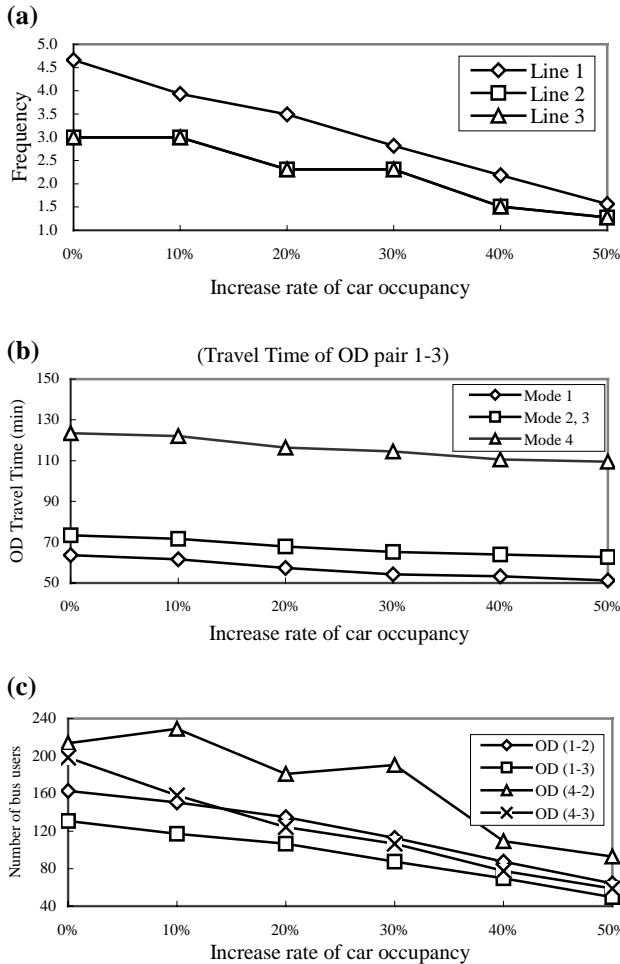


Figure 6: (a). Optimal frequency vs. car occupancy rate
 (b). Travel time OD (1-3) vs. car occupancy rate
 (c). Number of bus users vs. car occupancy rate

6. Conclusion

In this study, a new analytical framework for frequency design of bus transit in mixed traffic network with variable demand is explicitly given by considering the fact that road congestion equally increase the in-vehicle travel time for all modes on the same network. In case of bus transit, bus passenger flow is also included in addition to the traffic flow to compute the in-vehicle

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